▶ YONG CHENG, *The limit of incompleteness for Weak Arithmetics.*
School of Philosophy, Wuhan University, Wuhan 430072 Hubei, Peoples Republic of China.
*E-mail*: `world-cyr@hotmail.com`.

In this work, I examine the limit of incompleteness for Weak Arithmetics w.r.t. interpretation. For a recursively axiomatizable consistent theory $T$, I define that $\mathsf{G1}$ holds for $T$ iff for any recursively axiomatizable consistent theory $S$, if $T$ is interpretable in $S$, then $S$ is incomplete. My question is: can we find a weakest theory w.r.t. interpretation such that $\mathsf{G1}$ holds for it?

It is often thought that Robinson's theory $\mathbf{R}$ is such a weakest theory. Given theories $S$ and $T$, let $S \lhd T$ denote that $T$ interprets $S$ but $S$ does not interpret $T$ ($S$ is weaker than $T$ w.r.t. interpretation). A natural question is: can we find a theory $S$ such that $\mathsf{G1}$ holds for $S$ and $S \lhd \mathbf{R}$?

I positively answer this question and show that there are many examples of such a theory $S$ via two different methods. Two main theorems are: (1) for each recursively inseparable pair, there is a theory such that $\mathsf{G1}$ holds for it and it is weaker than $\mathbf{R}$ w.r.t. interpretation; (2) for any Turing degree $\mathbf{0} < \mathbf{d} < \mathbf{0}'$, there is a theory $U$ such that $\mathsf{G1}$ holds for $U$, $U \lhd \mathbf{R}$ and $U$ has Turing degree $\mathbf{d}$. As two corollaries, I answer a question from Albert Visser and show that there is no weakest theory below $\mathbf{R}$ w.r.t. Turing degrees such that $\mathsf{G1}$ holds for it.