▶ SATORU NIKI, AND PETER SCHUSTER, *On Scott's semantics for many-valued logic.*

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Scott [2] proposed abstract entailment relations for a semantics in ordered abelian groups of Łukasiewicz's many-valued logic. Urquhart [3] had a similar semantics. By Scott's entailments one can also represent ideal objects in abstract mathematics [1].

We now show that Scott's semantics fails to be sound for the bottom-to-top direction of Scott's rule $\rightarrow_2$, which was left out from Scott's proof [2, Theorem 3.1]. Indeed,

$$(A \rightarrow B) \rightarrow B \vdash A, B$$

is derivable by Scott's rules but invalid under some interpretation indexed by $[0, \infty)$. Urquhart [4, p. 35] used the same example to show that soundness would fail for his own semantics if one did not require that every formula have a least point of validity. No such request is made by Scott, as it would affect completeness of his semantics.

[1] JAN CEDERQUIST AND THIERRY COQUAND, *Entailment relations and distributive lattices*, **Logic Colloquium '98** (Prague, Czech Republic), (S.R. Buss et al., editors), A. K. Peters, Natick, MA, 2000, pp. 127–139.

[2] DANA SCOTT, *Completeness and axiomatizability in many-valued logic* (Univ. California, Berkeley), (L. Henkin et al., editors), **Proc. Sympos. Pure Math.** XXV, Amer. Math. Soc., Providence, RI, 1974, pp. 411–435.

[3] ALASDAIR URQUHART, *An interpretation of many-valued logic*, **Mathematical Logic Quarterly**, vol. 19 (1973), no. 7, pp. 111–114.

[4] ALASDAIR URQUHART, *Basic many-valued logic*, **Handbook of Philosophical Logic, volume 2** (D.M. Gabbay, F. Guenthner, editors), Kluwer, Dordrecht, 2001, pp. 249–295.