▸ CHRISTIAN ESPÍNDOLA, *Preservation theorems for strong first-order logics.*
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We solve an open problem dating back to 1977 mentioned in an article by John P. Burgess, namely "Descriptive set theory and infinitary languages". From the last paragraph:

"One large problem in the model theory of strong first-order languages remains open, which does not lend itself to abstract, descriptive-set-theoretic statement: Can we prove for, say, $\mathcal{L}_{\omega_1,G}$, that any sentence preserved under substructure (resp. homomorphic image) is equivalent to a universal (resp. positive) sentence?"

We answer positively the question providing preservation results for this particular game logic. These are generalizations of the theorems of Łoś-Tarski (resp. Lyndon) on sentences preserved by substructures (resp. homomorphic images). The solution, in ZFC, is then extended to several variants of strong first-order logic that do not satisfy the interpolation theorem; instead, the results on infinitary definability are used. Another consequence of our approach is the equivalence of the Vopěnka principle and a general definability theorem on subsets preserved by homomorphisms.

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