

- MARTA FIORI CARONES, ALBERTO MARCONE, PAUL SHAFER, GIOVANNI SOLDÀ, *Reverse Mathematics of some principles related to partial orders.*

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In this talk, we will study (some variations of) the following theorem, due to Rival and Sands ([3]) in the context of Reverse Mathematics:

THEOREM. (RS-po) *Let P be an infinite partial order of finite width k . Then there is an infinite chain C of P such that for every element $p \in P$, p is comparable with 0 or infinitely many elements of C .*

In particular, we show that ACA_0 , the third of the Big Five subsystems of \mathbf{Z}_2 , is enough to prove **RS-po**, although no reversal is known to hold. An interesting result is obtained by fixing the width of the partial order P : if $k = 3$, we prove that the theorem is equivalent to **ADS**, a combinatorial principle introduced by Hirschfeldt and Shore in [2], and a widely studied element of the “zoo below ACA_0 ” (a very good presentation of which is given for instance in [1]). Notably, this version of the theorem appears to be the first natural mathematical statement proven to be equivalent to **ADS**.

Finally, some partial results on a stronger version of **RS-po**, where we require comparability with 0 or *cofinitely* many elements of C , will be presented.

[1] DENIS R. HIRSCHFELDT, *Slicing the truth: on the Computable and Reverse Mathematics of Combinatorial Principles*, World Scientific, 2015.

[2] DENIS R. HIRSCHFELDT AND RICHARD A. SHORE, *Some Principles weaker than Ramsey’s Theorem for Pairs*, *The Journal of Symbolic Logic*, vol. 72 (2007), no. 1, pp. 171–206.

[3] IVAN RIVAL AND BILL SANDS, *On the Adjacency of Vertices to the Vertices of an Infinite Subgraph*, *Journal of the London Mathematical Society*, vol. s2-21 (1980), no. 3, pp. 393–400.