► ZOÉ CHATZIDAKIS, Notions of difference closures of difference fields.

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It is well known that a differential field K of characteristic 0 is contained in a differential field which is differentially closed and has the property that it K-embeds in every differentially closed field containing K. Such a field is called a differential closure of K, and it is unique up to K-isomorphism. In other words, prime models exist and are unique. The proof uses the fact that the theory of differentially closed fields of characteristic 0 is totally transcendental.

One can ask the same question about difference fields: do they have a difference closure, and is it unique? The immediate answer to both these questions is no, for trivial reasons: in most cases, there are continuum many ways of extending an automorphism of a field to its algebraic closure. Therefore a natural requirement is to impose that the field K be algebraically closed. Similarly, if the subfield of K fixed by the automorphism is not pseudo-finite, then there are continuum many ways of extending it to a pseudo-finite field, so one needs to add the hypothesis that the fixed subfield of K is pseudo-finite.

In this talk I will show by an example that even these two conditions do not suffice.

There are two (and more) natural strengthenings of the notion of difference closure, and we show that in characteristic 0, these notions do admit unique prime models over any algebraically closed difference field K, provided the subfield of K fixed by the automorphism is large enough.

In model-theoretic terms, this corresponds to the existence and uniqueness of a-prime or $\kappa\text{-prime models.}$

In characteristic p > 0, no such result can hold.