▸ INESSA PAVLYUK, SERGEY SUDOPLATOV, *On ranks for families of theories of finite abelian groups.*
Novosibirsk State Pedagogical University, Novosibirsk, Russia.
E-mail: inessa7772@mail.ru.
Sobolev Institute of Mathematics, Novosibirsk, Russia; Novosibirsk State Technical University, Novosibirsk, Russia; Novosibirsk State University, Novosibirsk, Russia.
E-mail: sudoplat@math.nsc.ru.

We continue to study families of theories of abelian groups [1] characterizing e-minimal subfamilies [2] for finite abelian groups by Szmielew invariants $\alpha_{p,n}$, β_p, γ_p, ε [4, 5], where p are prime numbers, $n \in \omega \setminus \{0\}$, as well as describing possibilities for the rank RS [2].

We denote by $\mathcal{T}_{A,\mathrm{fin}}$ the family of all theories of finite abelian groups.

THEOREM 1. *For any infinite family $\mathcal{T} \subseteq \mathcal{T}_{A,\mathrm{fin}}$ the following conditions are equivalent:* (1) *$\mathcal{T}$ is e-minimal;* (2) $\dim(\mathcal{T}) = 1$*, i.e., $\mathcal{T}$ does not have independent limit values for Szmielew invariants;* (3) *for any upper bound $\alpha_{p,n} \geq m$ or lower bound $\alpha_{p,n} \leq m$, for $m \in \omega$, there are finitely many theories in $\mathcal{T}$ satisfying this bound; having finitely many theories with $\alpha_{p,n} \geq m$, there are infinitely many theories in $\mathcal{T}$ with a fixed value $\alpha_{p,n} < m$.*

THEOREM 2. *Let α be a countable ordinal, $n \in \omega \setminus \{0\}$. Then there is a subfamily $\mathcal{T} \subset \mathcal{T}_{A,\mathrm{fin}}$ such that* $\mathrm{RS}(\mathcal{T}) = \alpha$ *and* $\mathrm{ds}(\mathcal{T}) = n$.

The families $\mathcal{T}$ for the proof of Theorem 2 have closures $\mathrm{Cl}_E(\mathcal{T})$ inside $\mathcal{T}_{A,\mathrm{fin}} \cup \mathcal{T}_{A,\mathrm{pf}}$, where $\mathcal{T}_{A,\mathrm{pf}}$ is the set of theories of pseudofinite abelian groups, and these closures are d-definable.

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