

► TADEUSZ LITAK, *Algebras for preservativity*.

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I overview algebraic aspects of our ongoing work with Albert Visser, both published [1] and unpublished, on systems of *constructive strict implication* a.k.a. *Lewis arrow*  $\rightarrow$  [2]. The main motivation to study such systems comes from their arithmetical interpretations, particularly in terms of  $\Sigma_1^0$ -*preservativity* [4, 5]. After providing algebraic semantics for the minimal system  $iA^-$ , we give examples of some pleasant applications. They include:

- An algebraic connection between the arithmetical notion of *extension stability* with the standard modal notion of a *subframe logic*, using Wolter’s notion of a *describable operation* [6].
- Examples of non-derivability proofs for simple consequences of the explicit scheme for de Jongh-Sambin fixpoints impossible in Kripke semantics.
- Wolter-Zakharyashev-style transfer of results and techniques for classical bimodal logics to their constructive  $\rightarrow$ -counterparts via a suitable variant of the Gödel-McKinsey-Tarski translation [7].
- A unifying perspective on generalizations of Kripke, Veltman and neighbourhood semantics.

[1] Tadeusz Litak and Albert Visser, *Lewis meets Brouwer: constructive strict implication*, *Indagationes Mathematicae*, A special issue “L.E.J. Brouwer, fifty years later”, vol. 29 (2018), no. 1, pp. 36–90, URL: <https://arxiv.org/abs/1708.02143>

[2] Clarence Irving Lewis, *A Survey of Symbolic Logic*, U. of California Press, 1918.

[3] Albert Visser, *Substitutions of  $\Sigma_1^0$ -sentences: explorations between intuitionistic propositional logic and intuitionistic arithmetic*, *Annals of Pure and Applied Logic*, vol. 114 (2002), pp. 227–271.

[4] Rosalie Iemhoff, *Preservativity logic: An analogue of interpretability logic for constructive theories*, *Mathematical Logic Quarterly*, vol. 49 (2003), pp. 230–249.

[5] Frank Wolter, *Lattices of Modal Logics*, PhD thesis, Fachbereich Mathematik, Freien Universität Berlin, 1993.

[6] Frank Wolter and Michael Zakharyashev, *On the relation between intuitionistic and classical modal logics*, *Algebra and Logic*, vol. 36 (1997), pp. 121–125.